Code: 13A54302 **R13**

B.Tech II Year I Semester (R13) Supplementary Examinations June 2017

MATHEMATICS – III

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

*****

1 Answer the following: (10 X 02 = 20 Marks)

(a) Write any two properties of beta function.

(b) Compute the value of $\Gamma(-1/2)$.

(c) Show that $p_1(x) = x$.

(d) State the orthogonal property of Bessels differential equation.

(e) Check whether u(x, y) = sin x coshy is harmonic function.

(f) Discuss about a Transformation w = z + c, where 'c' is complex constant.

(g) Evaluate $\int_C e^z dz$ where C is $|z| = 1$.

(h) Expand $f(z) = e^z$ in Taylor's series about z = 1.

(i) Find the residue at z =1 of the function $f(z) = \dfrac{z^2}{(z-1)(z-2)^2}$.

(j) State Cauchy's residue theorem.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\int_0^\infty x^{n-1} e^{-kx}\, dx = \dfrac{\Gamma n}{k^n} \quad (n > 0, k > 0)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} Sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta = \dfrac{\beta(m,n)}{2}$.

OR

3 Solve in Series the equation $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$

UNIT – II

4 (a) Prove that $\dfrac{d}{dx}\left[x J_n(x) J_{n+1}(x)\right] = x\left[J_n^2(x) - J_{n+1}^2(x)\right]$.

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

OR

5 State and prove the Rodrigues formula of Legendre Polynomials.

Contd. in page 2

UNIT – III

6 (a) State and prove the Cauchy- Riemann equations in polar form.

(b) If $f(z) = u + iv$ is Analytic function of z, find f(z) if $2u + v = e^{2x}\left[(2x + y)\cos 2y + (x - 2y)\sin 2y\right]$

OR

7 (a) Find the bilinear Transformation which maps the points $z = \infty, i, 0$ into the points $w = -1, -i, 1$.

(b) Discuss about the Transformation $w = z^2$

UNIT – IV

8 (a) Evaluate $\int_c \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where c is the circle $|z| = 1$.

(b) Verify cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1 + i, -1 + i$

OR

9 Find the Laurent series expansion of $\frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z + 2| < 5$

UNIT – V

10 (a) Evaluate $\int_c \frac{dz}{(z^2 + 4)^2}$,using residue theorem, where c: $|z - i| = 2$

(b) Determine the Residue of the function $f(z) = \frac{z + 1}{z^2 (z - 2)}$ at each pole.

OR

11 Show that $\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi a^2}{1 - a^2} \quad where \ \ a^2 < 1$

*****