Code: 15A54402

R15

B.Tech II Year II Semester (R15) Supplementary Examinations December 2017

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours | Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Find $\beta(2.5, 1.5)$.

(b) Compute $\Gamma(4.5)$.

(c) Compute $J_1(1)$

(d) $J_1(x) = \frac{1}{x}[xJ_1(x) - J_2(x)]$ use recurrence relation.

(e) Find the fixed points of the bilinear transformation $w = (z-1)/(z+1)$.

(f) Since the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic, the real and imaginary parts satisfies Cauchy-Riemann equations. Hence p = 2.

(g) Find Laurent series for $f(z) = \frac{1}{1-z^2}$ about $z_0 = 1$.

(h) Define removable singularity.

(i) Evaluate $\oint_C e^{1/z^2} dz$ where C is $|z| = 2$ traversed counterclockwise.

(j) Evaluate $\oint_C \frac{dz}{z^2(z+4)} dz$ where C is $|z| = 2$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.

(b) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

OR

3 (a) Find the value of $\Gamma\left(-\frac{1}{2}\right)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \cos^{\mathrm{n}} x$.

UNIT – II

4 Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta$ where n being integer.

OR

5 Find the value of $J_{\frac{1}{2}}(x)$.

UNIT – III

6 Find the analytic function $f(z) = u(r,\theta) + i\, v(r,\theta)$, given that $v(r,\theta) = r^2 \cos 2\theta - r\cos\theta + 2$.

OR

7 (a) Obtain the bilinear transformations which maps the points $z = \infty$, i, 0 into the points $w = 0$, i, ∞ respectively.

(b) Find the critical points of the transformation $w^2 = (z-a)(z-b)$.

UNIT – IV

8 Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$, using complex integration formula.

OR

9 If $0 < |z-1| < 2$ then express $(z) = \frac{z}{(z-1)(z-3)}$, in a series of positive and negative powers of $(z-1)$.

UNIT – V

10 Apply the calculus of residues evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$.

OR

11 (a) Evaluate $\int_C \frac{2z-3}{z^2+3z^2} dz$ where C is $|z| = 4$, traversed counterclockwise use residue theorem.

(b) Evaluate $\oint_C \frac{dz}{z^3(z+4)} dz$ where C is $|z+2| = 3$, traversed counterclockwise.

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